4.4 Heat Exchanger Examples

Heat exchangers are one of the simplest units in process industries – they can be found in almost every plant. As their name suggests, heat exchangers are used for energy exchange between at least two fluid phase (gas or liquid) streams, a hot and a cold stream. Heat exchangers are usually distributed parameter process systems, but we can build approximate lumped parameter models of them using finite difference approximations of their spatial variables (as in the method of lines approximation scheme). A heat exchanger can then be seen as a composite lumped parameter process system consisting of elementary dynamic units as is depicted in Figure 4.2.

4.4.1 Heat Exchanger Cells

A heat exchanger cell is a primitive dynamic unit which consists of two perfectly stirred (lumped) balance volumes (called lumps) connected by a heat...
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conducting wall. We shall call one of the lumps the hot \((j = h)\) side and the other one the cold \((j = c)\) side. The lumps with their variables are shown in Figure 4.2.(/a)

1. **Modeling assumptions**

   In order to obtain a simple model with only two state equations, the following simplifying modeling assumptions are used:

   1. Constant volume and mass hold-up in both of the lumps \(j = c, h\).
   2. Constant physico-chemical properties, such as
      - density: \(\rho_j\)
      - specific heat: \(c_{Pj}\)
      for both lumps, i.e. for \(j = c, h\).
   3. Constant heat transfer coefficient \((U)\) and area \((A)\).
   4. Completely observable states, i.e. \(y(t) = x(t)\).

2. **Conservation balances**

   The continuous time state equations of the heat exchanger cell above are the following energy conservation balances:

   \[
   \dot{T}_{co}(t) = \frac{v_c(t)}{V_c} (T_{ci}(t) - T_{co}(t)) + \frac{UA}{c_{Pc} \rho_c V_c} (T_{ho}(t) - T_{co}(t)) \tag{4.47}
   \]
   \[
   \dot{T}_{ho}(t) = \frac{v_h(t)}{V_h} (T_{hi}(t) - T_{ho}(t)) + \frac{UA}{c_{Ph} \rho_h V_h} (T_{co}(t) - T_{ho}(t)) \tag{4.48}
   \]

   where \(T_{ji}\) and \(T_{jo}\) are the inlet and outlet temperatures, \(V_j\) is the volume and \(v_j\) is the volumetric flow rate of the two sides \((j = c, h)\) respectively.

3. **System variables**

   The state vector is therefore composed of the two outlet temperatures:

   \[
   x_1 := T_{co}, \quad x_2 := T_{ho} \tag{4.49}
   \]

   There are a number of possibly time-dependent variables on the right-hand side of the above equations which may act as manipulable input variables or disturbances, depending on the measurement and actuator settings and on any additional modeling assumptions we may have. These are as follows:
• the inlet temperatures: $T_{ci}$ and $T_{hi}$,
• the volumetric flowrates: $v_c$ and $v_h$.

The special cases of the heat exchanger cell models are obtained by specifying assumptions on their variation in time. For every case, the output equation is

$$y(t) = h(x(t)) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(4.50)

### 4.4.2 LTI State-space Model of a Heat Exchanger Cell

#### 4. Additional modeling assumptions

In order to obtain a finite dimensional linear time-invariant model in each of the cases, the following additional assumptions are applied:

5. Constant volumetric flow rates.
6. Manipulable inlet temperatures.

#### 5. State equations

With assumptions 5 and 6 above, Equations (4.47)–(4.48) become the following finite dimensional LTI state equations:

$$\dot{x} = \begin{bmatrix} \frac{v_c}{V_c} - \frac{UA}{c_{p_c} p_c V_c} & \frac{UA}{c_{p_h} p_h V_h} \\ \frac{v_h}{V_h} - \frac{UA}{c_{p_h} p_h V_h} \end{bmatrix} x + \begin{bmatrix} \frac{v_c}{V_c} & 0 \\ 0 & \frac{v_h}{V_h} \end{bmatrix} u$$

(4.51)

with

$$u = \begin{bmatrix} T_{ci} \\ T_{hi} \end{bmatrix}$$

(4.52)

We may divide the state and input matrices in the above equations into additive terms related to the underlying mechanisms as follows. The state matrix term originating from energy transfer is

$$A^{(tr)} = \begin{bmatrix} -\frac{UA}{c_{p_c} p_c V_c} & \frac{UA}{c_{p_h} p_h V_h} \\ \frac{v_h}{V_h} - \frac{UA}{c_{p_h} p_h V_h} \end{bmatrix}$$

(4.53)

The input and output convection to the lumps gives rise to the following terms in the state and input matrices respectively:

$$A^{(oconv)} = B^{(iconv)} = \begin{bmatrix} \frac{v_c}{V_c} & 0 \\ 0 & \frac{v_h}{V_h} \end{bmatrix}$$

(4.54)

Using these matrices, we can write

$$A = A^{(tr)} - A^{(oconv)}, \quad B = B^{(iconv)}$$

(4.55)